Mathematical Medley
Problems Corner

Volume 33 No. 1 September 2006

The following problems are posed in Volume 32 No. 2, December 2005. We have not yet received satisfactory solutions so we will extend the deadline to 31 December 2006.

A. Prized Problems

1. Find all the real solution pairs (x, y) that satisfy the system

$$\frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{y}} = (x + 3y)(3x + y)$$

$$\frac{1}{\sqrt{x}} - \frac{1}{2\sqrt{y}} = 2(y^2 - x^2).$$

(Note: $\sqrt{x}$ denotes the *positive* square root of the real (nonnegative) number x).

Proposed by Albert Wong, Temasek Polytechnic.

2. (a) Prove that for any positive integer n,

$$\sum_{k=0}^{\lfloor n/4 \rfloor} \binom{n}{4k} = 2^{n-2} + (\sqrt{2})^{n-2} \cos \frac{n\pi}{4}$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

(b) Prove that

$$\sum_{k=1}^{45} \tan^2(2k-1)^\circ = 4005.$$

(Note: Solution by direct algorithmic computation will not be accepted)

B. Instruction

(1) Prizes in the form of book vouchers will be awarded to one or more received best solutions submitted by secondary school or junior college students in Singapore for each of these problems.

(2) To qualify, secondary school or junior college students must include their full name, home address, telephone number, the name of their school and the class they are in, together with their solutions.

(3) Solutions should be sent to : The Editor, Mathematics Medley, c/o Department of Mathematics, National University of Singapore, 2 Science Drive 2, Singapore117543 ; and should arrive before 31 December 2006.

(4) The Editor's decision will be final and no correspondence will be entertained.